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RELIABILITY AND STRUCTURAL INTEGRITY

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RELIABILITY AND STRUCTURAL INTEGRITY

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ABSTRACT

An analytic model is developed to calculate the reliability of a structure after it is inspected for cracks. The model accounts for the growth of undiscovered cracks between inspections and their effect upon the reliability after subsequent inspections. The model is based upon a differential form of Bayes' Theorem for reliability, and upon fracture mechanics for crack growth.

SYMBOLS

a,b	Half-length of crack
a_c	Critical half-length of crack
a_o	Threshold half-length of crack below which no cracks are detected
a_s	Half-length of crack whose probability of detection is specified to be 0.9
a^*	Half-length of crack which grows to infinity between inspections
B	Event "a crack is indicated"
B'	Event "a crack is not indicated"
$C_0, C_1, C_2, C_3, \beta_1, \beta_2$	Constants
$f[B\|a]$	Probability that a crack is indicated, given that it is present
$f[B'\|a]$	Equals $1 - f[B\|a]$
l	Final half-length of crack
R_{02}	Probability of surviving one period with no initial inspection
R_{12}	Probability of surviving one period with an initial inspection

R_{23} Probability of surviving one period, given that the structure passed inspection at t_s, survived one inspection period, then passed a second inspection

z Half-length of crack which grows to a_c between inspections

INTRODUCTION

Cracks occasionally appear in structures. They grow larger under repeated loads. If a structure is to remain strong, the cracks must be detected and repaired. The reliability of the structure will depend upon how many cracks are present, how long they are, and how well they can be detected so that they can be repaired.

Recent (unpublished) studies of crack detectability have established some probabilities of crack detection, given that a crack exists. The purpose of the present paper is to develop the methodology by which the reliability after inspection can be calculated from the reliability before inspection and the probability of crack detection. The method takes into account the variability of detection probability with crack size and the growth of cracks between inspections. The probability of multiple cracks in a structure is taken to be small compared with the probabilities of one crack or no crack; the multiple crack situations are beyond the scope of this paper.

CRACK SIZES AND DETECTION

In any structure, the crack population can be divided into two categories: potential fatigue cracks which have not initiated yet and which will not become detectably large for one or two orders of magnitude more flights than the second category of cracks; and cracks which were there initially, or which have already been initiated by cyclic or repeated stresses. A probability density function for cracks which represents both categories is

$$g(a) = \begin{cases} C_0\, \delta(a - o) + C_2\, \beta_2\, e^{-\beta_2 a} & \text{for } a \leq a_c \\ 0 & \text{for } a > a_c \end{cases} \tag{1}$$

where $\delta(a - o)$ is the Dirac delta function and C_0 is the fraction of cracks which have not yet initiated. (Note: Mathematically, C_0 is a normalizing constant.) It was assumed that no structures will appear for routine inspection if they already have cracks longer than the critical crack length, a_c. Under these conditions C_2 and β_2 are related.

$$C_2 = \frac{1 - C_0}{1 - e^{-\beta_2 a_c}} \qquad (2)$$

Consequently, the parameters C_0 and β_2 are independently adjustable to fit Equation (1) to data from actual experiences. A typical curve is sketched in Figure 1.

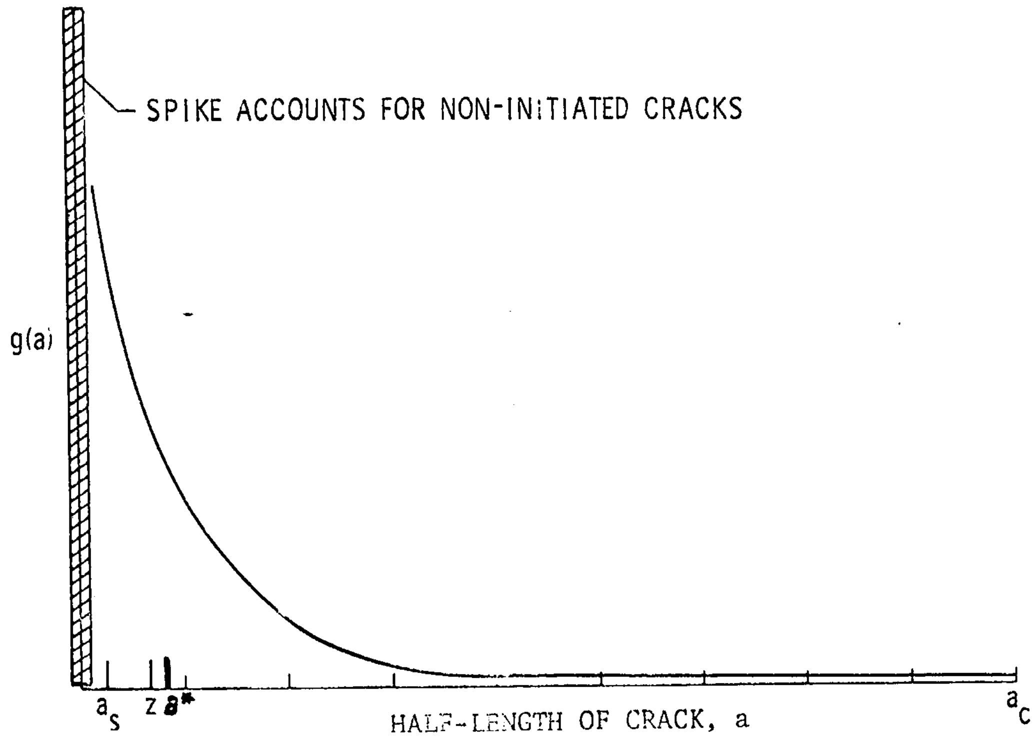

Figure 1. An illustration of the probability density function for flaw sizes. Most values of parameters gave functions which decreased much more rapidly than the curve shown.

Crack detectability varies with crack size. In general, during nondestructive inspection, large cracks are more easily found than small cracks. A detection function which represents the detectability, given that a crack is present, is

$$f(B/a) = \begin{cases} 0 & \text{for } a \leq a_o \\ C_1\left(1 - e^{-\beta_1(a-a_o)}\right) & \text{for } a \geq a_o \end{cases} \qquad (3)$$

where a_o is the threshold of detection, and $C_1 \leq 1$ is the asymptote for the probability of detection (see Fig. 2). The inequality, $C_1 < 1$, represents the fact that occasionally quite large cracks are overlooked.

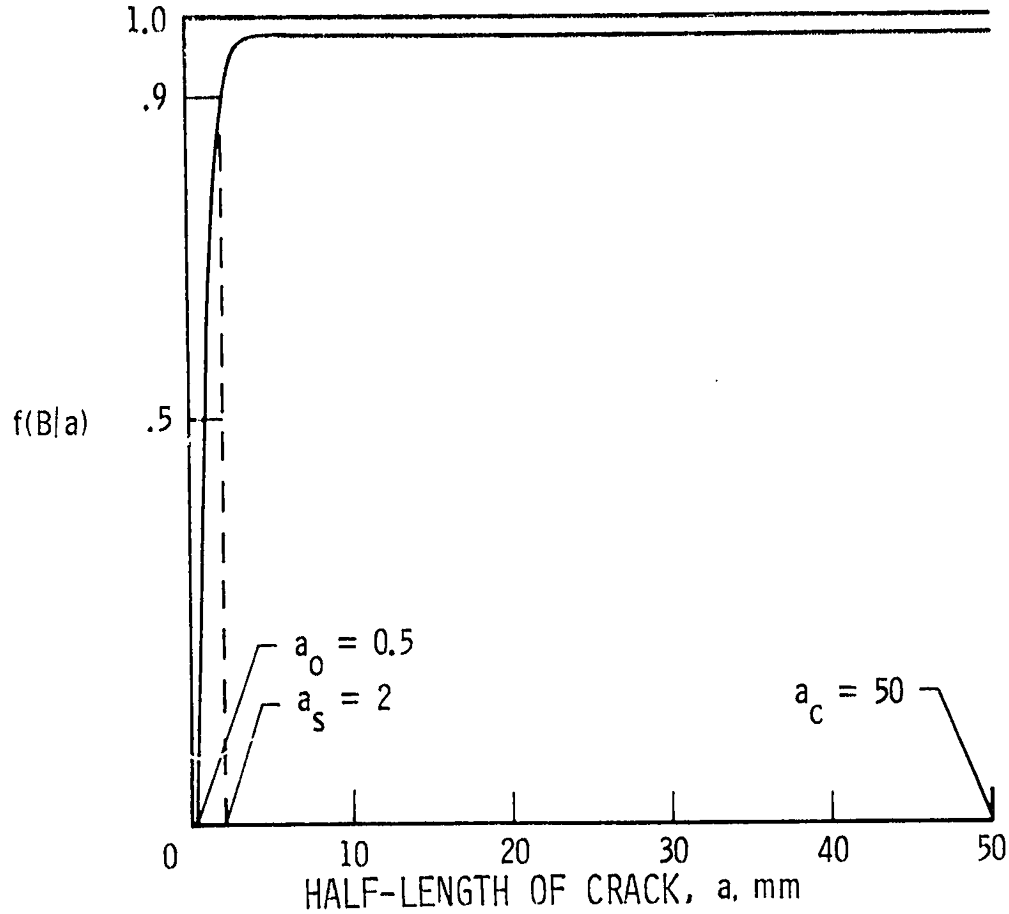

Figure 2. Probability a crack is discovered if it exists.

For this paper, the numerical values used for the probability of detection function were $C_1 = 0.98$; the threshold a_o was 0.5 mm; the crack length which was detectable with probability 0.9 was $a_s = 2.0$ mm; the critical crack length was $a_c = 50$ mm.

CRACK GROWTH AND INSPECTION PERIOD

Unrepaired cracks grow. From fracture mechanics, a simple crack growth expression is

$$\frac{da}{dF} = C_3 \, a^n \tag{4}$$

where F is the number of flights and C_3 and n are material constants. For aluminum, titanium, and aircraft steels $1 < n \leq 2$ [1]. One way to determine C_3 might be by flight-by-flight type laboratory tests. Equation (4) can be integrated,

$$\int_b^l \frac{da}{a^n} = \int_0^F C_3 \, dy$$

from which

$$\frac{1}{l^{(n-1)}} = \frac{1}{b^{(n-1)}} - (n - 1)C_3 F \tag{5}$$

where F is the number of flights between inspections, a is the crack length just after the jth inspection, and l is the length just before the $(j+1)^{th}$ inspection. Define "m" as the number of intervals of length $(t_c - t_s)/m$ needed for a crack to grow from a_s at time t_s to a_c at time t_c (see Fig. 3); then

$$F = \frac{1}{mC_3(n-1)} \left[\frac{1}{a_s^{\,n-1}} - \frac{1}{a_c^{\,n-1}}\right] = \frac{\alpha}{C_3(n-1)} \tag{6}$$

where

$$\alpha = \frac{1}{m}\left[\frac{1}{a_s^{\,n-1}} - \frac{1}{a_c^{\,n-1}}\right]$$

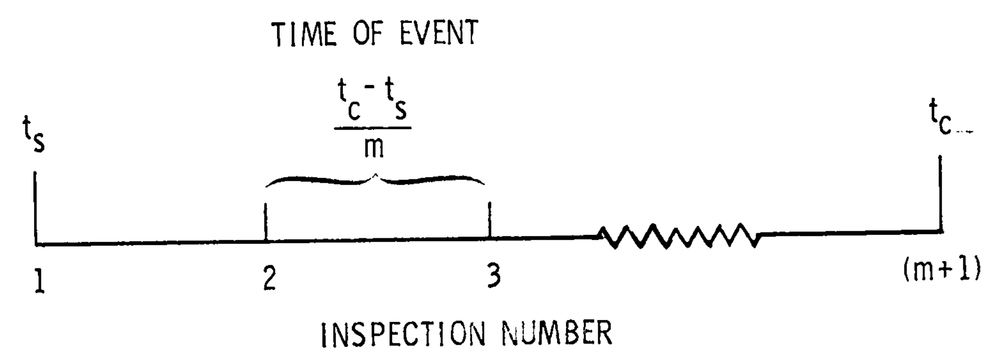

Figure 3. Relations among times, inspection periods, and inspection numbers.

From Equations (5) and (6)

$$\frac{1}{l^{n-1}} = \frac{1}{b^{n-1}} - \alpha \tag{7}$$

Define z as the size of the crack which will grow to a_c during one inspection period.

$$\frac{1}{z} = \left[\frac{1}{a_c^{\,n-1}} + \alpha\right]^{\frac{1}{n-1}} \tag{8}$$

Also, let a^* be the value of "b" at which the right-hand side of Equation (7) becomes zero and "z" becomes infinite.

$$a^* = \left(\frac{1}{\alpha}\right)^{\frac{1}{n-1}} \tag{9}$$

Figure 1 shows how a_s, z, a^*, and a_c are ordered. Figure 4 shows how z and a^* vary with the number of inspections, m, between t_s and t_c.

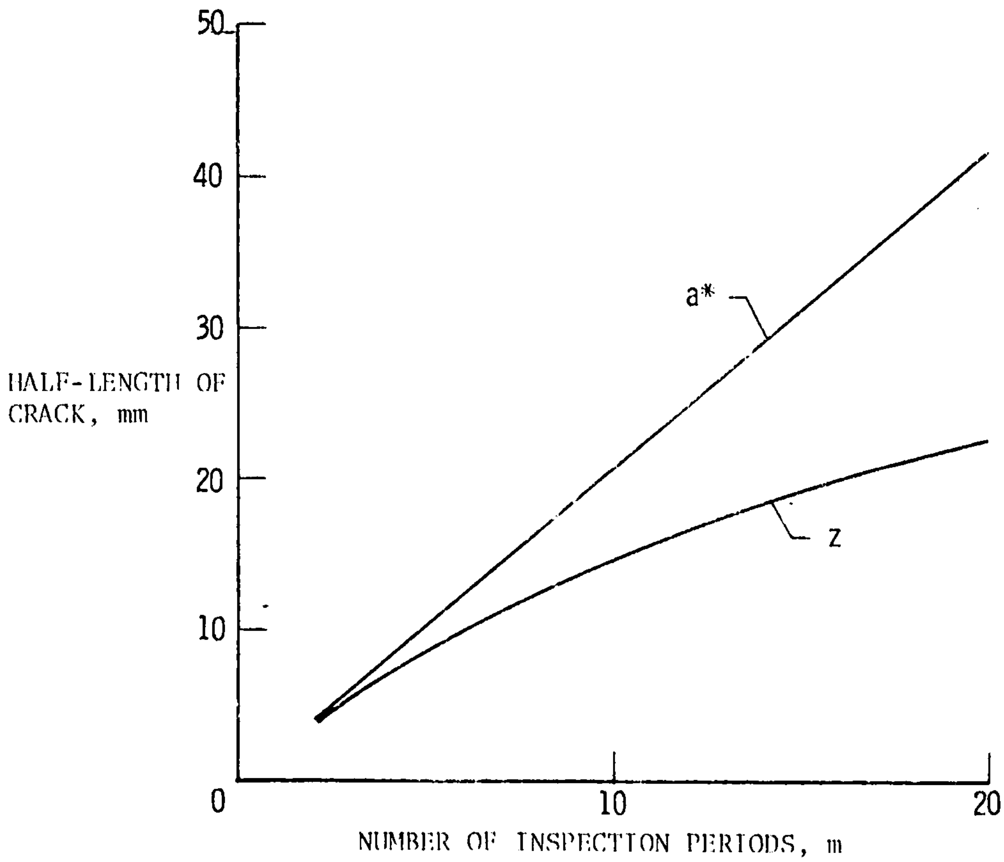

Figure 4. Crack, z, which reaches a_c between inspections; and crack, a^*, which reaches infinity between inspections.

ANALYSIS FOR RELIABILITY

The unreliability, or probability that a structure will not last from one inspection to the next, equals the probability that an undiscovered crack of length "b_1" (where $z < b_1 \leq a_c$) remains in the structure at the start of the period between inspections; such a crack will grow to exceed a_c before the next inspection.

The probability that the structure will last through one inspection interval without a previous inspection is

$$R_{01} = 1 - \int_z^{a_c} g(a)da$$

$$= 1 - C_2 \left(e^{-\beta_2 z} - e^{-\beta_2 a_c}\right) \tag{10}$$

However, the reliability if inspected will depend upon the probability density function (p.d.f.) for undiscovered cracks. This p.d.f. can be found from Bayes' Theorem [2]. The p.d.f. for undiscovered cracks is

$$h[a/B'] = \frac{f[B'|a]g(a)}{\int_0^{a_c} f[B'|a]g(a)da} \tag{11}$$

where the prime indicates "not." The reliability after the first inspection is

$$R_{12} = 1 - \int_z^{a_c} h[a|B']da$$

$$= 1 - \zeta_1/\xi_1 \tag{12}$$

where

$$\zeta_1 = C_2(1 - C_1)\left(e^{-\beta_2 z} - e^{-\beta_2 a_c}\right)$$

$$+ \frac{C_1 C_2 \beta_2}{(\beta_1 + \beta_2)} \left[e^{-(\beta_1+\beta_2)z+\beta_1 a_o} - e^{-(\beta_1+\beta_2)a_c+\beta_1 a_o}\right]$$

and

$$\xi_1 = C_0 + C_2\left(1 - e^{-\beta_2 a_o}\right) + C_2(1 - C_1)\left(e^{-\beta_2 a_o} - e^{-\beta_2 a_c}\right)$$

$$+ \frac{C_1 C_2 \beta_2}{(\beta_1 + \beta_2)}\left[e^{-\beta_2 a_o} - e^{-(\beta_1+\beta_2)a_c + \beta_1 a_o}\right]$$

Not all of the cracks which were undiscovered during the first inspection will cause failure; most cracks were shorter than z. But now, during the period between the first and second inspection, these overlooked or undetectable cracks will grow, and some may become longer than z; if these are overlooked during the second inspection, they will grow to exceed a_c before the third inspection. Consequently, the unreliability (probability of not surviving until the third inspection) is the integral from z to a_c of a new distribution function which represents grown cracks. This function can be obtained from $h[a|B']$ [3]:

$$h_2[l|B'] = h_2(l) = \frac{h(Q|B')\,\frac{dQ}{dl}\,f(B'|l)}{\int_0^{a_c} h[Q|B']\,\frac{dQ}{dl}\,f[B'|l]\,dl} \tag{13}$$

$$= \frac{\Gamma}{\xi_2}$$

where

$$Q = \left[\left(\frac{1}{l}\right)^{n-1} + \alpha\right]^{-\left(\frac{1}{n-1}\right)} \tag{14}$$

Specifically,

$$\Gamma = \begin{cases} \left[C_0\delta(l-0) + C_2\beta_2 e^{-\beta_2 Q}\right]\frac{dQ}{dl} & \text{for } 0 \le l \le a_o \\ C_2\beta_2 e^{-\beta_2 Q}\left(1 - C_1 + C_1 e^{-\beta_1(l-a_o)}\right)\frac{dQ}{dl} & \text{for } a_o \le l \le \left[\left(\frac{1}{a_o}\right)^{n-1} - \alpha\right]^{-\left(\frac{1}{n-1}\right)} \\ C_2\beta_2 e^{-\beta_2 Q}\left(1 - C_1 + C_1 e^{-\beta_1(Q-a_o)}\right)\left(1 - C_1 + C_1 e^{-\beta_1(l-a_o)}\right)\frac{dQ}{dl} & \\ & \text{for} \left[\left(\frac{1}{a_o}\right)^{n-1} - \alpha\right] \le l \le a_c \\ 0 & \text{for } l > a_c \end{cases}$$

This function does not account for structures in which cracks grew longer than a_c before the second inspection; these structures were regarded as having removed themselves from further consideration. The reliability of those structures which survive until – and then pass – the second inspection is

$$R_{23} = 1 - \int_z^{a_c} h_2(l)\,dl$$

$$= 1 - \zeta_2/\xi_2 \tag{15}$$

where

$$\zeta_2 = C_2(1 - C_1)^2\left(e^{-\beta_2 Q(z)} - e^{-\beta_2 a_c}\right)$$

$$+ \frac{C_1C_2\beta_2(1 - C_1)}{(\beta_1 + \beta_2)}\, e^{\beta_1 a_o}\left(e^{-(\beta_1+\beta_2)Q(z)} - e^{-(\beta_1+\beta_2)z}\right)$$

$$+ C_1(1 - C_1)C_2\, e^{\beta_1 a_o}\left(e^{-\beta_2 Q(z)-\beta_1 z} - e^{-(\beta_1+\beta_2)a_c}\right)$$

$$+ \frac{C_1^{\,2}C_2\beta_2}{(\beta_1 + \beta_2)}\left(e^{-\beta_1 z-(\beta_1+\beta_2)Q(z)} - e^{-\beta_1 a_c-(\beta_1+\beta_2)z}\right)e^{2\beta_1 a_o}$$

$$- C_1(1 - C_1)C_2\beta_2\, e^{\beta_1 a_o}\int_z^{a_c} e^{-\beta_2 Q-\beta_1 l}\,dl$$

$$- \frac{C_1^{\,2}C_2\beta_1\beta_2}{(\beta_1 + \beta_2)}\, e^{2\beta_1 a_o}\int_z^{a_c} e^{-\beta_1 l-(\beta_1+\beta_2)Q}\,dl$$

$$\xi_2 = C_0 + C_2\left(1 - e^{-\beta_2 Q(a_o)}\right) + C_2(1 - C_1)\left(e^{-\beta_2 Q(a_o)} - e^{-\beta_2 a_o}\right)$$

$$+ C_1 C_2\left(e^{-\beta_2 Q(a_o)} - e^{-\beta_1 y(a_o)+(\beta_1-\beta_2)a_o}\right)$$

$$+ C_2(1 - C_1)^2\left(e^{-\beta_2 a_o} - e^{-\beta_2 z}\right)$$

$$+ \frac{C_2\beta_2 C_1(1 - C_1)}{(\beta_1 + \beta_2)} e^{\beta_1 a_o}\left(e^{-(\beta_1+\beta_2)a_o} - e^{-(\beta_1+\beta_2)z}\right)$$

$$+ C_2 C_1(1 - C_1)\left(e^{-\beta_2 a_o - \beta_1(y(a_o)-a_o)} - e^{-\beta_2 z - \beta_1(a_c - a_o)}\right)$$

$$+ \frac{C_2 C_1^2 \beta_2}{(\beta_1 + \beta_2)}\left(e^{-\beta_1 y(a_o)-(\beta_1+\beta_2)a_o} - e^{-\beta_1 a_c-(\beta_1+\beta_2)z}\right) e^{2\beta_1 a_o}$$

$$- C_1 C_2 \beta_1 \int_{a_o}^{y(a_o)} e^{-\beta_2 Q - \beta_1(l-a_o)}\, dl$$

$$- C_2 C_1(1 - C_1)\beta_1 \int_{y(a_o)}^{a_c} e^{-\beta_2 Q - \beta_1(l-a_o)}\, dl$$

$$- \frac{C_2\beta_2 C_1^2 \beta_1}{(\beta_1 + \beta_2)} e^{2\beta_1 a_o} \int_{y(a_o)}^{a_c} e^{-(\beta_1+\beta_2)Q-\beta_1 l}\, dl$$

where

$$y(x) = \left[\left(\frac{1}{x}\right)^{n-1} - \alpha\right]^{-\left(\frac{1}{n-1}\right)}$$

RESULTS AND DISCUSSION

Figure 4 contains some implications which help formulate a game plan for the design and use of a structure. Suppose, for some reason, m is chosen to be 2. All cracks larger than $z = 3.85$ mm (half-crack length) must be detected if the structure is to survive until the next inspection. If such a crack length has too low a probability of detection by the nondestructive inspection procedure, little will be gained by choosing a material with higher fracture toughness (crack growth rate is reasonably independent of fracture toughness); at best, even if the material can be made infinitely tough any undiscovered crack larger than $a^* = 4.17$ mm will still cause failure. Instead, the frequency of inspection should be increased (m chosen larger) so that the curves for z and a^* separate (see Fig. 4).

Figure 5 illustrates some typical relationships among the reliabilities under various conditions. Higher reliabilities are associated with short inspection periods (large number of inspections between t_s and t_c, Fig. 3). For some values of C_0, β_2, and m the reliability after the second inspection can be lower than the reliability after the first inspection; this happens when the probability of a crack which will grow to exceed z in one period is higher than the probability that a crack exists whose length is between z and a_c at the time of the first inspection. The analysis is modeling the real life situation where a crack is so likely to propagate in a structure that its reliability decreases with age.

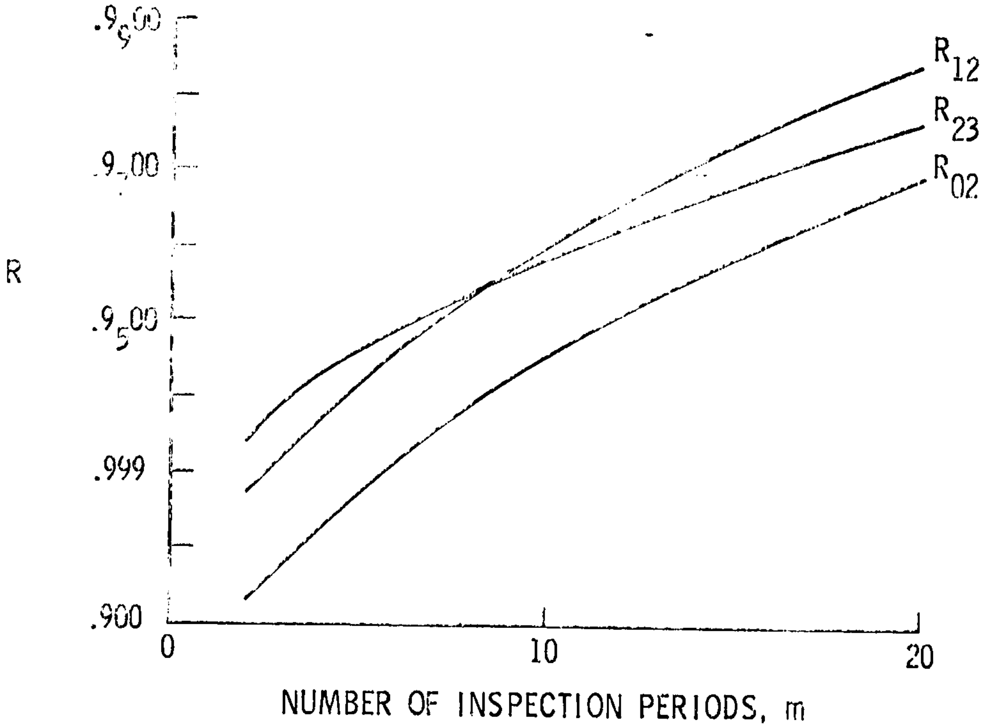

Figure 5. Reliabilities before and after the time of the first inspection (R_{01} and R_{12}) and after ($C_0 = .2$; P[crack between a_s and a_c before first inspection] = .2) second inspection (R_{23}). See test for values of parameters.

High reliabilities after inspections are always associated with structures whose reliabilities were high before inspection. Under certain conditions this implies that an optimum reliability might be obtained by a nonuniform spacing of inspections; in particular, three inspections might be better distributed as two independent inspections at t_s and one inspection two periods later rather than three equally spaced inspections starting at t_s. Additional calculations confirmed this.

CONCLUDING REMARKS

An analytic model for reliability was developed which contained the salient features of practical situations where inspection procedures are less than absolutely perfect, where crack detectability is a function of crack length, and where undiscovered cracks grow larger and influence the reliability of succeeding inspections. The analysis can be used to study the effects of various schemes for material choice and inspection intervals.

The relationships between the crack length which grows to detectable length between inspections, the detectability, and the frequency of inspections shows that inspection frequencies may be increased to compensate for imperfect nondestructive inspection procedures.

Calculations also indicate that the optimum reliability may be obtained from inspection schedules which are not uniformly spaced.

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[3]Emanuel Parzen, "Modern Probability Theory and Its Applications," John Wiley and Sons, 1962.